

Spurious Effects in Perturbative Calculations

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Abstract: We show spurious effects in perturbative calculations due to different orderings of inhomogenous terms while computing corrections to Green functions for two different metrics. These effects are not carried over to physically measurable quantities like the renormalized value of the vacuum expectation value of the stress-energy tensor.

KEY WORDS: Green function; vacuum expectation value

Introduction

Finding logarithmic behaviour of correlation functions instead of pure power law has always been interesting in quantum field theory. QCD differs from a free theory at the asymptotic region only by its logarithmic corrections. The presence of these corrections differentiate a physically interesting theory from a trivial model. Another effect is the possibility that logarithmic corrections in perturbation theory may sum up to an anomalous power law, as seen in the Thirring model [1]. Keeping these examples in mind, it is always get exciting to encounter logarithmic behaviour while calculating Green functions. That is why one may be curious whether encountering such terms always points to important physical phenomena.

Here we want to study the interaction of a gravitational field described by a certain metric with a scalar field. We do not know how to quantize the gravitational field, though. This fact necessitates the use of semi-classical methods . These methods treat the gravitational field classically and couples it to the scalar field only by writing the d'Alembertian in the background of that metric [2]. This operation reduces the full field theoretical problem to an external field calculation. We study the n-point functions of the problem and try to deduce information about the full theory from these quantities. Out of these n-point functions, the two point function, which is the Green function of the d'Alembertian, is the most important one, since by differentiating it we can get the vacuum expectation value, VEV, of the stress-energy tensor, $T_{\mu\nu}$.

To be able to calculate renormalized value of the VEV of the stress-energy tensor, $\langle T_{\mu\nu} \rangle_{ren}$, we have to find an algorithm to regulate the divergences of the two-point function, G_F , at the coincidence limit. The initial step in this programme is to study the singularity structure of G_F for the particular model studied. We know that the singularity at the coincidence limit of the Green function for a free scalar field in flat space is quadratic. At this point one is confronted with a very important theorem [3] concerning the singularity behaviour of Green functions in different backgrounds. This theorem states that if a metric is flat at any region in space-time, the singularities of the Green

function in the background of this metric should exhibit Hadamard behaviour [4], which is at worst quadratic. Although we may by-pass the results of this theorem by studying metrics that are not C^∞ , still it is very improbable for Green functions of scalar particles in the background metrics that are flat at any region in space-time to have singularities that are worse than quadratic.

The theorem quoted above makes us suspicious of any extra logarithmic terms we encounter while calculating the Green functions in the background of different metrics, especially if the calculation is performed perturbatively. We want to see if such terms are genuine or the result of the different regularization procedures used, arising when we make our expressions unnecessarily singular. Whether such effects are genuine or not can be checked by comparing perturbative results with the exact ones, when available, and studying different ways of grouping terms with more or less singular behaviour.

Here we are going to continue our investigation of spurious effects in Green function calculations using different metrics [5]. We will see that while we take alternative routes in solving inhomogenous partial differential equations results in solutions with different singularity structure for the Green's function, this difference cancels out in the calculation of physical quantities like the VEV of the stress-energy tensor, after it is renormalized.

Our model is an impulsive gravitational plane wave solution of Hogan [6].

$$ds^2 = -2 \cos^2 aU |dz + \Theta(U) \frac{\tan aU}{a} \overline{H}(\overline{z}) d\overline{z}|^2 + 2dUdV, \quad 1$$

with the only one non-vanishing Ricci tensor component,

$$R_{44} = -2a^2 \Theta(U), \quad 2$$

and only one non-vanishing Weyl tensor component ,

$$\Psi_4 = H(z) \delta(U). \quad 3$$

Here Θ is the Heavyside unit step function, δ is the Dirac delta function, a is a constant and H is the z derivative of a smooth arbitrary function of z , $H(z) = \frac{dG}{dz}$. U and V are

null coordinates, $z = \frac{1}{\sqrt{2}}(x + iy)$. This metric is flat for $U < 0$ and conformally flat for $U > 0$. This metric is similar to the metric found by Nutku earlier [7] .

We have two reasons for studying this model . First, we have the exact result to compare with the perturbative one, in one of the two cases studied. Second, we know that the $\langle T_{\mu\nu} \rangle_{ren}$ should be null ^{/8,9}, a result we have to obtain at the end.

We see that this metric has a dimensional parameter a . We have seen that the presence of a dimensional parameter in the metric may result in non-standard (non-Hadamard) behaviour in the Green function , when this quantity is calculated perturbatively ^{/10}. Here we find that this change in the singularity structure occurs if we group our terms in a certain manner in the perturbation expansion even in the first order calculation. It is absent for the exact calculation . It is amusing, however, to see that these terms are absent if we group the inhomogenous terms in a different manner. This fact shows that the change in the singularity in the former calculation is due to the wrong choice of the parameters, which results in severer singularities, and, when regulated in the Schwinger formalism, ends up with terms which are worse than the other case by logarithms. We find that this is another method for generating logarithmic singularities in the coincidence limit for the Green functions at will . A similar phenomenon was shown to exist in [10], by taking the homogenous solutions into account. These singularities do not survive when the VEV of the renormalized stress-energy tensor, a measurable physical quantity, is calculated.

We first solve the Green function, G_F for a special $H(z)$ used in the metric given by eq. [1] exactly and point to the Hadamard behaviour of this expression. Then we get the perturbative solution in two ways, and show two different results obtained for G_F , only one of them in the Hadamard form. In the third section we use a more complicated form of $H(z)$, where we could not obtain the exact result and show the same conflicting behaviour when G_F is calculated perturbatively. At the end we note that when the VEV of the stress-energy tensor is computed the different expressions for G_F give the same result for $\langle T_{\mu\nu} \rangle_{ren}$. All through our work we use conformal coupling.

2.1 Exact Calculation for the First Metric

We choose $G = az$ for the arbitrary function in the metric given above, which gives the following expression for the d'Alembertian operator written in the background of this metric

$$\begin{aligned} \square = 2 \frac{\partial^2}{\partial U \partial V} - 2a \tan(2aU) \frac{\partial}{\partial V} + \frac{4}{\cos^2(2aU)} (\sin(2aU) - 1) \frac{\partial^2}{\partial x^2} \\ - \frac{4}{\cos^2(2aU)} (\sin(2aU) + 1) \frac{\partial^2}{\partial y^2}. \end{aligned} \quad 4$$

If we write

$$\square f(x, y, U, V) = 0, \quad 5$$

we find

$$f = \frac{e^{iRV} e^{ik_1 x} e^{ik_2 y}}{\cos^{1/2}(2aU)} e^{\frac{k_1^2 - k_2^2}{iRa \cos(2aU)}} e^{-\frac{k_1^2 + k_2^2}{iRa} \tan(2aU)}. \quad 6$$

Here k_1, k_2, R are the Fourier modes that will be integrated over in the Green's function calculation. These integrations are performed easily and we get

$$G_F = - \frac{a (\Theta(U - U') - \Theta(U' - U))}{8\sqrt{2}i\pi^2 \sin 2a(U - U') \left((V - V') - a \frac{(x-x')^2}{4\Delta^2} - a \frac{(y-y')^2}{4\Delta_1^2} \right)}, \quad 7$$

where

$$\Delta^2 = \left(-\frac{1}{\cos(2aU)} + \tan(2aU) + \frac{1}{\cos(2aU')} - \tan(2aU') \right), \quad 8$$

$$\Delta_1^2 = \left(\frac{1}{\cos(2aU)} + \tan(2aU) - \frac{1}{\cos(2aU')} - \tan(2aU') \right). \quad 9$$

We see easily that in the coincidence limit this function has a quadratic divergence, same as in the flat metric.

2.2 Perturbative Calculation for the first metric

We expand the operator \square in powers of a . If we write

$$\square = L_0 + aL_1 + \dots, \quad 10$$

we find

$$L_0 = 2 \frac{\partial^2}{\partial U \partial V} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}, \quad 11$$

$$L_1 = 2U \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right). \quad 12$$

We also expand both the eigenfunction and the eigenvalue of the equation

$$\square \phi = \lambda \phi \quad 13$$

in powers of a as $\phi = \phi^0 + a\phi^1 + \dots$ and $\lambda = \lambda^0 + \lambda^1 + \dots$. We find

$$\phi^0 = \frac{1}{(2\pi)^2 (2|R|)^{\frac{1}{2}}} e^{i\left(\frac{K}{2R}U + RV + k_1x + k_2y\right)}, \quad 14$$

$$\lambda^0 = K - k_1^2 - k_2^2, \quad 15$$

$$\lambda^1 = (\phi^0, L_1 \phi^0) = 0. \quad 16$$

Here k_1, k_2, K, R are the different modes we have to integrate over to find the Green function G_F . ϕ^1 satisfies the equation

$$(L_0 - \lambda_0)\phi^1 + L_1\phi^0 = 0. \quad 17$$

The form of this equation suggests the ansatz

$$\phi^1 = \phi^0(g(U, z) + h(U, \bar{z})) \quad 18$$

where $z = \frac{1}{\sqrt{2}}(x + iy)$. This ansatz yields the following equation for the unknown functions g and h :

$$iR \left(\frac{\partial g}{\partial U} + \frac{\partial h}{\partial U} \right) - \frac{i}{\sqrt{2}}(k_1 + ik_2) \frac{\partial g}{\partial z} - \frac{1}{\sqrt{2}}(k_1 - ik_2) \frac{\partial h}{\partial \bar{z}} + (k_1^2 + k_2^2)U = 0. \quad 19$$

We separate this equation into two parts, one for a function of z and U , and the other a function of \bar{z} and U . One choice for this decomposition is taking

$$iR \frac{\partial g_1}{\partial U} - \frac{1}{\sqrt{2}}(k_1 + ik_2) \frac{\partial g_1}{\partial z} + \frac{1}{2}(k_1^2 + k_2^2)U = 0, \quad 20$$

and

$$iR \frac{\partial h_1}{\partial U} - \frac{i}{\sqrt{2}}(k_1 - ik_2) \frac{\partial h_1}{\partial \bar{z}} + \frac{1}{2}(k_1^2 + k_2^2)U = 0. \quad 21$$

At this point a clarification is in order. Since the inhomogeneous term is a constant, as far as z and \bar{z} are concerned, there is ambiguity how it is shared among the two equations. Here we designate by g_1, h_1 , the particular choice for the decomposition given above.

Once this separation is made, the integration is immediate. We find

$$g_1 = -\frac{(k_1^2 + k_2^2)}{i\sqrt{2}(k_1 + ik_2)}Uz - iR \frac{(k_1^2 + k_2^2)}{2(k_1 + ik_2)^2}z^2, \quad 22$$

$$h_1 = \frac{i(k_1^2 + k_2^2)}{\sqrt{2}(k_1 - ik_2)}U\bar{z} - iR \frac{(k_1^2 + k_2^2)}{2(k_1 - ik_2)^2}\bar{z}^2. \quad 23$$

To calculate the Green function G_F we have to sum over all the modes,

$$G_F = -\sum_{\lambda} \frac{\phi\phi^*}{\lambda}, \quad 24$$

which reduces in our first order calculation to

$$G_F^{(1)} = -\sum \frac{\phi_0\phi_0^*[g_1 + g_1^* + h_1 + h_1^*]}{\lambda_0}. \quad 25$$

When written explicitly, we get

$$G_F^{(1)} = -\int_{-\infty}^{\infty} dR \int_{-\infty}^{\infty} dK \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2 \times \\ \times \frac{e^{(i[R(V-V') + \frac{K}{2R}(U-U') + k_1(x-x') + k_2(y-y')])}}{(2\pi)^4(2|R|)(K - k_1^2 - k_2^2)} (F(\chi) + F^*(\chi')), \quad 26$$

where

$$F = -i \frac{(k_1^2 - k_2^2)}{k_1^2 + k_2^2} \left((k_1x + k_2y)U + \frac{R}{2(k_1^2 + k_2^2)} [(k_1^2 - k_2^2)(x^2 - y^2) + 4k_1k_2xy] \right), \quad 27$$

and χ is the generic name for the four variables U, V, x, y . We use the Schwinger representation to write

$$\frac{1}{K - k_1^2 - k_2^2} = i \int_0^{\infty} d\alpha \exp(-i(K - k_1^2 - k_2^2)\alpha - \alpha\delta), \quad 28$$

$$\frac{1}{(k_1^2 + k_2^2)^2} = - \int_0^\infty d\beta \beta \exp(-i(k_1^2 + k_2^2)\beta - \gamma\beta) \quad 29$$

where δ and γ are infinitesimal positive and real quantities. We perform the integrals in the usual manner. The end result is written using an infrared parameter m and the zeroth order Hankel function $H_0^{(2)}$ as

$$G_F = \frac{1}{4\sqrt{2}(2\pi)^2} \left(\frac{-A}{B\Delta} + \frac{i\pi A}{B^2} H_0^{(2)}(2m\sqrt{\Delta}) + \dots \right), \quad 30$$

where ... contains terms with the same singularity behaviour as the first two, i.e. terms with quadratic divergence and quadratic divergence times a logarithmic divergence in the coincidence limit. The logarithmic divergence is given by the $H_0^{(2)}$ term which goes to a logarithm as its argument goes to zero when the infrared cut-off is removed,

$$H_0^{(2)} \propto \log \Delta + \log 2m, \quad 31$$

and we discard the $\log 2m$ term. In the above expression

$$A = (x - x')(Ux - U'x') - (y - y')(Uy - U'y'), \quad 32$$

$$B = (x - x')^2 + (y - y')^2, \quad 33$$

$$\Delta = 2(U - U')(V - V') - (x - x')^2 - (y - y')^2. \quad 34$$

In this expression we see that the expected Hadamard behaviour, i.e. the worse divergence being only quadratic, is modified by a logarithm.

It is amusing to note that this behaviour, which is not reflected to the exact solution, is an artefact of the choice we used in separating our eq. [19] into the holomorphic and antiholomorphic parts. Another choice is separating eq. [19] as

$$iR \frac{\partial g_2}{\partial U} - \frac{i}{\sqrt{2}}(k_1 + ik_2) \frac{\partial g_2}{\partial z} + \frac{(k_1 + ik_2)^2}{4} U = 0, \quad 35$$

$$iR \frac{\partial h_2}{\partial U} - \frac{1}{\sqrt{2}}(k_1 - ik_2) \frac{\partial h_2}{\partial \bar{z}} + \frac{(k_1 - ik_2)^2}{4} U = 0. \quad 36$$

The integrations of these equations give immediately

$$g_2 = \frac{-i}{2\sqrt{2}}(k_1 + ik_2)Uz - iR \frac{z^2}{4}, \quad 37$$

$$h_2 = \frac{-i}{2\sqrt{2}}(k_1 - ik_2)U\bar{z} - iR\frac{\bar{z}}{4}. \quad 38$$

Note that with the latter choice for decomposition of the equation, we have reduced the powers of $(k_1 + ik_2), (k_1 - ik_2)$ in the denominator. Since we integrate these expressions from minus infinity to plus infinity, terms in the denominator vanish in this range and we use the Schwinger prescription given above to regulate them. The calculation of G_F with less severe divergences is much simpler now. The Green function integration is straightforward. For $U > U'$, it reads

$$G_F = \frac{1}{4\sqrt{2}(2\pi)^2} \times \left(\frac{C_1[(x - x')(Ux - U'x') - (y - y')(Uy - U'y')] + C_2[(u - u')(x^2 - y^2 - x'^2 + y'^2)]}{\Delta^2} \right) \quad 39$$

where C_1, C_2 are two constants. This singularity has the same singularity behaviour as the exact solution at the coincidence limit.

3. Second metric

Here we show the same thing with a different function $G(z)$ used to specify the metric in eq. [1] explicitly, to illustrate that the phenomena we find is not special to only one choice of the trial function H . We take the next simplest form, $G = \frac{az^2}{2}$. Then the metric reads, in the region where it is not flat,

$$ds^2 = 2dUdV - 2 \left[dzd\bar{z} (\cos^2(aU) + a^2 z\bar{z} \sin^2(aU)) + \frac{a}{2} \sin(2aU) (z(d\bar{z})^2 + \bar{z}(dz)^2) \right]. \quad 40$$

We find

$$\begin{aligned} \square = & -\frac{2a \cos(aU) \sin(aU)(1 + z\bar{z})}{B} \frac{\partial}{\partial V} + 2 \frac{\partial^2}{\partial U \partial V} + \frac{2a \cos^3(aU) \sin(aU)}{B^3} \left(\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}} \right) \\ & + \frac{2a \cos(aU) \sin(aU)}{B^2} \left(z \frac{\partial^2}{\partial z^2} + \bar{z} \frac{\partial^2}{\partial \bar{z}^2} \right) - \frac{2a^2 \cos^2(aU) \sin^2(aU)}{B^2} \left(z \frac{\partial}{\partial z} + \bar{z} \frac{\partial}{\partial \bar{z}} \right) \\ & - 2 \frac{\cos^2(aU) + a^2 z\bar{z} \sin^2(aU)}{B^3} \frac{\partial^2}{\partial z \partial \bar{z}}. \end{aligned} \quad 41$$

Here $B = \cos^2(aU) - a^2 z \bar{z} \sin^2(aU)$. We could not solve the Green function of this operator exactly; so, we do not know the exact singularity structure of it. We compare, however, the singularity structure of the two expressions we obtain by grouping the inhomogeneous terms differently. Just as in the first example, one way results in a function with Hadamard behaviour, the other gives rise to a term which is modified by logarithmic corrections.

We expand the operator \square in powers of a . At the first nontrivial order we get

$$\square \approx 2 \left(\frac{\partial^2}{\partial U \partial V} - \frac{\partial^2}{\partial z \partial \bar{z}} \right) + 2a^2 U \left(-\frac{\partial}{\partial V} + \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}} + z \frac{\partial^2}{\partial \bar{z}^2} + \bar{z} \frac{\partial^2}{\partial z^2} \right). \quad 42$$

We separate \square into two parts $\square \approx L_0 + a^2 L_1$ and expand both the eigenvalue λ and the eigenfunction ϕ of the equation

$$\square \phi = \lambda \phi \quad 43$$

in the same manner. The zeroth-order and first-order eigenvalues and zeroth-order eigenfunction are as given in eq.s (14)- (16). We get the equivalent of eq. [17], with new L_1 for ϕ_1 . The structure of this equation again suggests the ansatz of eq. [18], $\phi_1 = \phi_0[g(U, z) + h(U, \bar{z})]$ which gives rise to the equation

$$\begin{aligned} & iR \frac{\partial}{\partial U} (g + h) - \left(\frac{ik_1 - k_2}{\sqrt{2}} \right) \frac{\partial g}{\partial z} - \left(\frac{ik_1 + k_2}{\sqrt{2}} \right) \frac{\partial h}{\partial \bar{z}} \\ & + U \left[z \left(\frac{ik_1 - k_2}{\sqrt{2}} \right)^2 + \bar{z} \left(\frac{ik_1 + k_2}{\sqrt{2}} \right)^2 - \sqrt{2} ik_1 - iR \right] = 0. \end{aligned} \quad 44$$

Here we get two different results depending on how we separate this equation into two equations for the two unknown functions. One choice is to write

$$iR \frac{\partial g_1}{\partial U} - \left(\frac{ik_1 - k_2}{\sqrt{2}} \right) \frac{\partial g_1}{\partial z} + \frac{Uz}{2} (ik_1 - k_2)^2 - \frac{iRU}{2} + \frac{iUk_1}{\sqrt{2}} = 0, \quad 45$$

$$iR \frac{\partial h_1}{\partial U} - \left(\frac{ik_1 + k_2}{\sqrt{2}} \right) \frac{\partial h_1}{\partial \bar{z}} + \frac{U\bar{z}}{2} (ik_1 + k_2)^2 - \frac{iRU}{2} + \frac{iUk_1}{\sqrt{2}} = 0. \quad 46$$

These equations are integrated to get

$$g_1 = \frac{iRz^3}{6} + \left[\frac{(ik_1 + k_2)^2 R^2}{2(k_1^2 + k_2^2)^2} + \frac{\sqrt{2}(ik_1 + k_2)^2 k_1 R}{2(k_1^2 + k_2^2)^2} - \frac{\sqrt{2}}{4} U(ik_1 - k_2) \right] z^2$$

$$+ \frac{U(iR - i\sqrt{2}k_1)(ik_1 + k_2)}{k_1^2 + k_2^2} z \quad 47$$

and the corresponding expression for h_1 where the z is replaced by \bar{z} and $ik_1 + k_2$ goes into $ik_1 - k_2$ and vice versa. If we use this solution to obtain the Green function, we obtain functions with quadratic singularity at the coincidence point as well as functions whose singularities are modified by a logarithmic term. It is also amusing that the logarithmic behaviour comes out only when we have an ambiguity in separating the equation. The first, second, fourth and the fifth terms have only the quadratic Hadamard singularity, whereas the third and the sixth terms, in this form, give rise to logarithms. The details of this calculation can be found in reference [11]. We want to state only that when we calculate the Green function for the term that reads

$$2k_1 R \frac{(ik_1 + k_2)}{(k_1^2 + k_2^2)^2} z^2 \quad 48$$

plus the \bar{z} part, we get

$$\frac{(U - U')(x - x')}{((x - x')^2 + (y - y')^2)^2} (x^2 + x'^2 - y^2 + y'^2 + 2xy + 2x'y') H_0^{(2)}(2m\Delta) \quad 49$$

plus terms with the same singularity structure in addition to terms with only quadratic singularity structure. Δ in this expression was defined in eq. [34]. We can modify the third and the sixth term in eq. [47] to reduce the power of the terms in the denominator, though, and write the differential equations as

$$R \frac{\partial g_2}{\partial U} - \frac{(k_1 + ik_2)}{\sqrt{2}} \frac{\partial g_2}{\partial z} - \frac{Uz}{2i} (k_1 + ik_2)^2 - \frac{RU}{2} - \frac{U}{\sqrt{2}} (k_1 + ik_2) = 0, \quad 50$$

$$R \frac{\partial h_2}{\partial U} - \frac{(k_1 - ik_2)}{\sqrt{2}} \frac{\partial h_2}{\partial \bar{z}} - \frac{U\bar{z}}{2i} (k_1 - ik_2)^2 - \frac{RU}{2} - \frac{U}{\sqrt{2}} (k_1 - ik_2) = 0. \quad 51$$

In this expression we keep most of the terms same as those given in eq.[45] and change only one term, the only term which gave the logarithmic correction.

The solution of the above equations reads

$$g_2 = \frac{iRz^3}{6} + iz^2 \left(\frac{U(k_1 + ik_2)}{\sqrt{2}} - \frac{R^2}{2(k_1 + ik_2)^2} + \frac{\sqrt{2}iR}{2(k_1 + ik_2)} \right) - zU \left(1 + \frac{iR}{\sqrt{2}(k_1 + ik_2)} \right) \quad 52$$

and the similar expression for $h_2(U, \bar{z})$. We check the behaviour in the coincidence for parts of G_F that are replaced in the new expression. The terms that are changed are the fourth and the fifth terms in eq. [52]. We can show that these new terms give rise to only to quadratic divergence in G_F . The calculation of the fifth term gives rise to

$$\frac{2(u - u')}{((x - x')^2 + (y - y')^2)\Delta} \left((x - x')(x^2 + x'^2 - y^2 - y'^2) + 2(y - y')(xy + x'y') \right) \quad 53$$

with no logarithm. The calculation of the fourth term gives only the Δ term in the denominator and exhibits Hadamard behaviour.

4. Conclusion

We have already noted [10] that while solving eq. [17] after making the ansatz $\phi_1 = \phi_0(g(z, U) + h(\bar{z}, U))$, we are solving an equation of the type

$$\left[R \frac{\partial}{\partial U} - \frac{(k_1 + ik_2)}{\sqrt{2}} \frac{\partial}{\partial z} \right] g = I \quad 54$$

where I denotes the inhomogeneous part of the equation. This equation also has a solution for the case when $I = 0$, the homogeneous case. Indeed, any arbitrary function with the argument $\frac{U}{R}(k_1 + ik_2) + \sqrt{2}z$ is a solution. If the function f has a different dimension as compared to ϕ_0 , then we have to multiply the solution to the homogenous equation by a dimensional constant. In our problem the only such quantities are the modes, R and $k_1 + ik_2$ that exist in eq. [14]. This seems to be completely innocent, as far as g is concerned. The fact that we have to sum over all the modes, however, changes the result obtained for G_F . One shows that we can generate $H_0^{(2)}$, the function that modifies the Hadamard behaviour in this way. Which one of the two factors, R or $k_1 + ik_2$, is used does not change the character of the new singularity structure. It was shown in [10] that only $H_0^{(2)}$ survives the two derivatives if we want to extract the vacuum expectation value of the stress-energy tensor, $\langle T_{\mu\nu} \rangle$ from G_F using the established methods^{/2}. It is also shown that this new singularity structure which already exists in the homogeneous solution, and which is here unsurfaced by taking different combinations for the inhomogeneous term, does not change the result that plane waves do not polarize the vacuum^{/8,9}.

Here we show another way to generate such spurious effects. We refer to our earlier work ^{/10} to establish that the renormalized value of the VEV of the stress-energy tensor is independent of all these spurious effects. The essence of the argument is that there is no finite part of the resulting expression when the coincidence limit is taken, since we have only an isolated pole divergence, and no remaining finite part.

We see that there is more than one way to generate spurious singularities for perturbatively computed Green functions. Here we study examples which are obtained by grouping terms in different manners in inhomogenous differential equations. It is a relief that these spurious effects do not change the value calculated for $\langle T_{\mu\nu} \rangle_{ren}$.

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REFERENCES

1. K.G.Wilson, Phys. Rev **D2**, (1970) 1473 and 1478 ;
- 2 N.D.Birrell and P.C.W. Davies, *Quantum Fields in Curved Space Space*, Cambridge University Press, Cambridge (1982) , S.A. Fulling, *Aspects of Quantum Field Theory in Curved Space-Time* Cambridge University Press, Cambridge, 1989, R.M. Wald, *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*, The University of Chicago Press, Chicago, 1994;
3. S.A.Fulling, M.Sweeney and R.Wald, Comm. Math. Phys. **63** (1978) 257;
4. J.Hadamard,*Lectures on Cauchy's Problem in Linear Partial Differential Equations*, Yale University Press, New Haven (1923) ;
5. Earlier work can be found in M.Hortaçsu and R. Kaya , Gen. Rel. Grav. **30** (1998) 1645;
6. P.A.Hogan, 1997 " Plane gravitational Waves and Bertotti- Robinson Space-Times", University College Dublin , preprint;
7. Y.Nutku in *Directions in General Relativity, Papers in honor of Charles Misner* ed. by B.L.Hu, M.P.Ryan Jr. and C.V. Vishveshara, Cambridge University Press , Cambridge (1993);
8. S.Deser, J. Phys. A: Math. Gen. **81** (1975) 972;
9. G.W.Gibbons, Comm. Math. Phys. **45** (1975) 191;
10. M.Hortaçsu and K.Ülker, , Class. Quantum Grav., **15** (1998) 1415;
11. B.C.Lütfüoğlu, I.T.U. Thesis, unpublished.